# A LOW-FREQUENCY SAW-TOOTH GENERATOR

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A LOW-FREQUENCY SAW-TOOTH GENERATOR

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## I. INTRODUCTION

In the modern high-vacuum cathode-ray tube, science has available an excellent instrument for the measurement and observation of phenomena of an electrical nature or of such a nature that it can readily be translated into electrical or magnetic effects. Before it can be conveniently employed in such a fashion, the cathode-ray tube must be incorporated in an oscilloscope circuit which supplies the necessary accelerating and focusing voltages, and which also provides a deflection voltage or sweep, sometimes called a "time base." This sweep voltage must vary with respect to time in some known fashion, perhaps sinusoidally or exponentially, but the most generally useful sweep is one which varies linearly with time. Although spiral or circular sweeps are sometimes used, the sweep which moves the fluorescent spot linearly across the screen leads to a trace or pattern which is much more easily interpreted, and, further, entails considerably less in the way of circuit complications.

Since the introduction of the first practicable low-voltage, gas-focused cathode-ray tube by Van der Bijl and Johnson in 1922, a great deal of work has been done by many scientists toward the development of linear sweep circuits or saw-tooth generators. The interests of most of these workers, however, have been centered on the higher frequency sweep circuits, and, while much work has been done toward extending the upper frequency limit, there is practically nothing to be found in the literature with regard to sweep generators capable of operating at frequencies of the order of a few cycles per second. The average oscilloscope, although it is usually said to have a sweep which is linear down to a frequency of thirty cycles per second, shows sufficient distortion at sixty cycles per second to be objectionable, and at a frequency of thirty cycles per second, the time base is so nonlinear as to be practically useless on a quantitative basis. Although mechanical oscillographs may be used in the observation of low-frequency phenomena, the moving elements of such devices have many times the mass of the electron beam in a cathode-ray tube, and, consequently, lack the ability to exhibit, with any degree of accuracy, the high-speed transients which are frequently superimposed on low-frequency phenomena.

A low-frequency saw-tooth wave which is quite linear may be generated by means of a rotary potentiometer driven at a constant speed by some mechanical device. The design of such a saw-tooth generator, however, entails considerable complication, both mechanical and electrical, if provision is made for proper synchronization and frequency control.

It is most desirable to be able to observe and measure transients occurring on alternating-current power lines. In the study of servo-mechanisms, the ability to observe phenomena which are inherently low-frequency in nature would be most advantageous. Numerous other cases arise in which it is desirable to have a saw-tooth generator capable of operating below the frequency range of the ordinary oscilloscope, e.g., in the observation of phenomena which occur at a low repetition rate. Many possible applications exist in medical science in such fields as electrocardiography and electro-encephalography.

Due to the apparent need for a linear low-frequency sweep circuit to be used with the conventional oscilloscope, this investigation was undertaken to produce a saw-tooth generator capable of operating over the range from one cycle per second to thirty cycles per second.

## II DISCUSSION

Since the horizontal deflection amplifier of the average oscilloscope has a rapidly falling response below thirty cycles per second, it appeared essential that the saw-tooth generator be capable of either driving directly the horizontal deflection plates of the cathode-ray tube, or that it be equipped with an amplifier capable of amplifying the lowest frequency desired. In this work it was found that it is quite possible with a relatively simple circuit to produce a saw-tooth voltage wave of such a magnitude as to provide full deflection of the electron beam when applied directly to the deflection-plate terminals of the cathode-ray tube. This voltage, however, is inherently unbalanced, i.e., it varies in only one polarity with respect to ground. If the electron beam of the ordinary cathode-ray tube is deflected by grounding one deflection plate and applying a voltage to the other plate, the spot is both defocused and distorted at the end of the trace. To overcome this difficulty it is necessary to drive the deflection plates symmetrically with respect to ground, and the driving voltage should be balanced. This can be accomplished quite readily in one of two ways. The sweep circuit can be made up of two saw-tooth generators operated "back to back," or the saw-tooth voltage may be used to drive a push-pull amplifier which in turn supplies the deflection voltage to the cathode-ray tube.

The two simplest and best known means for producing a voltage which varies linearly with time are to apply a constant voltage to an inductance and to employ the potential difference which appears across a small series resistance, or to charge or discharge a capacitor through some sort of resistance. Both of these methods, however, produce a voltage which is inherently exponential in form. The instantaneous voltage across a small resistance, R', placed in series with a resistance-inductance series circuit across which a voltage is suddenly applied at time, $t = 0$, may be expressed by the equation:

$$e = iR' = R' \frac{E}{R} (1 - \varepsilon^{-\frac{Rt}{L}}) \tag{1}$$

$e$ = instantaneous voltage across R'.

$i$ = instantaneous current.

$E$ = applied voltage.

$R$ = total resistance of circuit.

$R'$= small series resistance.

$L$ = inductance.

$t$ = time.

$\varepsilon$ = a constant = 2.718

If, after steady state conditions are reached, the applied voltage is suddenly removed by short-circuiting, the instantaneous voltage across R' becomes

$$e = R' \frac{E}{R} \varepsilon^{-\frac{Rt}{L}} \tag{2}$$

The instantaneous voltage across the capacitor of a resistance-capacitance series circuit to which a voltage is suddenly applied at time, $t = 0$, is:

$$e = E(1 - \varepsilon^{-\frac{t}{RC}}) \tag{3}$$

where C is the capacitance, and the other notation is the same as that used in Equation 1. If, after steady state conditions are reached, the applied voltage is suddenly removed by short-circuiting at time, $t = 0$, the expression for the voltage across the capacitor becomes:

$$e = E \varepsilon^{-\frac{t}{RC}} \tag{4}$$

Unfortunately, from the point of view of producing a saw-tooth wave, all of these voltages are inherently exponential in form. If only a small portion of the voltage wave produced by one of the above methods is used, the difference between it and a truly linear variation is quite small. This, however, necessitates the use of an excessively high supply voltage if a reasonable voltage variation is required, and has the additional disadvantage in low-frequency work that capacitances or inductances of unreasonable size are required.

An alternative is to employ some means for linearizing the voltage wave. This can be accomplished in a number of ways, but these generally involve the use of one of the following devices:[1]

(a) The inverse curvature of a tube characteristic.

(b) An auxiliary time constant.

(c) Negative Feedback.

In this work, it was decided from practical considerations by way of simplicity of design to employ a capacitor charged through a thyratron and discharged through what is effectively a very high resistance as a source of sweep voltage. The discussion will henceforth be limited to this type of circuit.

From Equation 4 it is seen that:

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[1]O. S. Puckle, Time Bases, p. 83.

$$e = E\varepsilon^{-\frac{t}{RC}}$$

By Ohm's law, we can write the equation for the instantaneous discharge current of the capacitor:

$$i = \frac{e}{R} = \frac{E}{R}\varepsilon^{-\frac{t}{RC}} \tag{5}$$

The derivative of e with respect to time is an expression for the rate of change of e, and, if the voltage wave is to be linear, this must be equal to a constant. Taking this derivative, the following equation results:

$$\frac{de}{dt} = -\frac{1}{C}\frac{E}{R}\varepsilon^{-\frac{t}{RC}} \tag{6}$$

which becomes, upon combining with Equation 5:

$$\frac{de}{dt} = -\frac{1}{C}i \tag{7}$$

It is thus seen that, if the discharge current can be caused to remain constant, a linear sweep voltage will result.

Actually, there is no known means for drawing an absolutely constant current from a variable voltage source. It is possible, however, through the proper application of negative feedback, to reduce the current variation, and, hence the variation in the rate of change of voltage to any desired degree. It is true that the resulting voltage wave will still be of an exponential form. On the other hand, it has been shown[2] that the other methods of linearization also produce a voltage wave which is

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[2]Arthur C. Clarke, "Linearity Circuits," Wireless Engineer, No. 249, Vol. XXI, June 1944, pp. 256-266.

exponential in form. Since the other methods of linearization require more care in adjustment and are more subject to difficulties due to such things as strong magnetic fields and variations in temperature and tube characteristics, it was decided to employ negative feedback to linearize the sweep.

The circuit which was finally developed is shown in Diagram I-A. The capacitor is charged to very nearly the supply voltage through a type 2051 thyratron, not shown. When the thyratron is extinguished, the discharge cycle starts, with the capacitor discharging through V-2, a type 6AC7/1852 pentode connected as shown. At the beginning of the discharge cycle, the voltage across the capacitor has its maximum value, and, consequently, the discharge current is a maximum. This current in flowing through the cathode resistance of V-2 produces a voltage drop which results in a voltage being applied to the grid of V-1, also a type 6AC7/1852 pentode. Due to the presence of V-3, a type OA3/VR-75 voltage-regulator tube, in the cathode circuit of V-1, the cathode of V-1 is maintained at a potential 75 volts positive with respect to the negative bus. Since the other circuit components are so chosen that the grid of V-1 is maintained negative with respect to the cathode, V-1 operates as a conventional amplifier. The plate of V-1 is connected to the grid of V-2 through V-4, a type OC3/VR-105 voltage-regulator tube, which serves simply as a coupling device, making it possible to obtain the proper d-c voltage at the grid of V-2, so that the grid is maintained negative with respect to the cathode. Then, insofar as V-1 is concerned, V-2 operates as a cathode follower.

As the capacitor is discharged, the voltage decreases, and the discharge current tends to decrease. Any decrease in discharge current, however, causes a decrease in the voltage drop across the resistance R-7 + R-8 and results in a negative signal being applied to the grid of V-1. This causes a decrease in the current through V-1 and produces a positive signal at the grid of V-2, which, in turn, increases the discharge current to very nearly its initial value. The change, $i_1$, in discharge current can be expressed:

$$i_1 = \frac{e\ A_2}{\mu_2\ R_{L2}\ (A_1\ A_2 + 1)} \tag{8}$$

$e$ = change in capacitor voltage.

$i_1$ = change in discharge current.

$\mu_2$ = amplification factor of V-2.

$R_{L2}$ = cathode resistance of V-2.

$A_1$ = amplification of V-1 stage.

$A_2$ = amplification of V-2 stage considered as a cathode follower.

From this expression, derived in Appendix I, the percentage variation in discharge current, which is numerically equal to the percentage of non-linearity of the voltage wave produced across the capacitor, can readily be calculated. Although the effects of R-3 and R-5 are ignored in the derivation of Equation 8, these effects are negligible since these resistors are for the purpose of suppressing parasitic oscillations and are of the order of a few ohms. This variation can be held down to any desired value by the simple expedient of using a high-$\mu$ tube for V-2 and providing sufficient amplification in the feedback amplifier. Since the circuit is direct coupled throughout, there is no lower limit to the frequency of operation insofar as the discharge circuit is concerned. Furthermore, the difficulties usually encountered in d-c amplifiers due to small changes in component values and tube characteristics present no problem here since their effects are greatly reduced by the large amount of negative feedback present.

The current through the discharge circuit is plotted as a function of capacitor voltage for different values of resistance in the cathode circuit of V-2 in Curves II-A through II-E. The data for these curves were obtained by placing a milliammeter in series with the discharge circuit and replacing the capacitor with a variable-voltage supply. Hence, these curves are accurate for any frequency down to zero cycles per second.

In the development of this circuit, the first circuit used was one in which the screen of V-2 was returned to the positive supply bus, and the cathode of V-1 was returned to the negative bus through a resistor. Data were taken on the variation of discharge current with capacitor voltage, using this circuit, and this information is plotted as Curves I-A and I-B. It can readily be seen that the variation over the working range is quite noticeable, being of the order of 5 per cent. This variation was found to be due partially to degeneration caused by the resistance in the cathode circuit of V-1.

After replacing the resistance in the cathode of V-1 with the voltage-regulator tube, V-3, additional data were taken, which were plotted as Curve I-C. It can be seen that, although this curve is slightly improved, there is still a marked variation in discharge current over the selected working range from 150 to 325 volts.

At this point one significant fact was noticed. It is well known that, as the plate potential of a pentode falls below the screen potential the screen current increases quite rapidly. Indeed, there is a marked variation in the screen current until the plate potential is well above the screen potential. In this case it was noted that the increase in plate or discharge current of V-2 with increasing capacitor voltage was accompanied by an almost identical decrease in screen current. Actually, the feedback circuit was holding the current through the cathode resistance of V-2 very nearly constant, but the discharge current through the plate circuit of V-2 was allowed to increase as the screen current decreased. This difficulty was overcome by connecting the screen of V-2 to the plate through a voltage dropping resistance, thus making the current flowing to the screen as well as to the plate of V-2 serve to discharge the capacitor. Then all current flowing through the cathode resistance of the tube, V-2, is capacitor discharge current. This connection results in V-2 having a plate characteristic somewhat between those of 6AC7 pentode and the same tube connected as a triode. The amplification factor is higher than it would be were the tube operated as a triode, and this is quite desirable from a constant current standpoint as is indicated by Equation 8. These changes resulted in the final circuit as shown in Diagram I.

Fine frequency control is accomplished by varying the resistance in the cathode of V-2. The manner in which the discharge current varies with cathode resistance is readily seen from Curve III. In order to avoid non-linearity due to leakage currents, it is essential that the discharge current through V-2 not be allowed to fall below approximately one milliampere. Since good quality capacitors were used in the experimental model and the capacitors were operated well below their rated voltage, it was found that the discharge current could be reduced to three quarters of a milliampere with no noticeable non-linearity due to the leakage currents. This value of current corresponds to a cathode resistance of 82000 ohms, and this is the maximum value which was used. It is seen from Curve III that, as the cathode resistance is reduced, the discharge current increases more and more rapidly. For a given capacitor and a fixed voltage swing, the time required to discharge the capacitor from the initial voltage to the final voltage is:

$$T = \frac{Ce}{i} \tag{9}$$

T = time of discharge through a fixed voltage change.

C = capacitance.

e = voltage change.

i = discharge current, assumed constant.

If we ignore the time lost in the recharging process, and if the capacitor is recharged immediately upon being discharged to the lower voltage limit, the frequency of operation may be expressed as:

$$f = \frac{1}{Ce}\, i \tag{10}$$

From Equation 10, it is seen that the frequency of operation varies directly as the discharge current. With this in mind, it is readily seen from Curve III that, if the discharge current is allowed to rise above six milliamperes, the frequency will be considerably affected by slight changes in cathode resistance. Thus, if the cathode resistance is decreased below about 12000 ohms it becomes quite difficult to set the frequency at any given value or to synchronize the sweep. In addition, a further difficulty is encountered as the discharge current becomes large. This discharge current is continuous through the charge cycle as well as the discharge cycle. In order to extinguish the thyratron, V-5, at the end of the charge cycle, the cathode of V-5 must rise at least to very nearly the plate potential and remain there long enough for the tube to deionize. If the discharge current is high, the potential of the cathode of the thyratron, which is connected to one side of the sweep capacitor, falls below the anode potential before the tube has deionized. When this occurs, the thyratron operates continuously, and the sweeping process stops. To overcome this difficulty, the capacitor, C-1, was connected between the cathode of V-5 and the grid of V-1. This results in the grid of V-1 being driven positive during the charge cycle, and in turn applies a negative pulse to the grid of V-1, which reduces the discharge current to a small value long enough to permit the thyratron to deionize. It should be noted that this results in some variation in discharge current early in the discharge cycle. If the values of C-1 and R-4 are carefully chosen, the time constant may be made such that only a very small portion of the discharge cycle is affected.

Even with the discharge current limited to the range of from one to six milliamperes, a frequency range of six to one can be covered. Coarse frequency adjustment is accomplished by switching sweep capacitors. By using a one-microfarad and a six-microfarad capacitor, it is possible to cover all frequencies from one cycle per second to thirty cycles per second.

It should be noted that the capacitors used for this saw-tooth generator are quite small compared to those employed in more conventional sweep circuits when the frequency range covered in taken into consideration. Upon inspection of Equation 10, the reason for this is quite apparent. In most sweep generators, the change in voltage across the sweep capacitor is made quite small compared to the supply voltage, in order to preserve linearity. It is readily seen that, if e is small, C must be made large to produce a given low frequency with a fixed value of discharge current. Due to the type of discharge circuit used in the saw-tooth generator under consideration, it is possible to make the change in voltage quite large compared to the supply voltage. The lower limit of voltage across the capacitor is fixed by the voltage at which the current through V-2 begins to vary appreciably with capacitor voltage. As indicated by Curve II, this is approximately 125 volts for high values of discharge current. At this point the grid of V-1 begins to draw current, and the feedback circuit is rendered ineffective, permitting the current through V-2 to fall off quite rapidly with capacitor voltage. The actual lower limit of the working range of capacitor voltage is set by the adjustment of R-11 which determines the potential at the control grid of V-5. As soon as the discharging of the capacitor drops the potential of the cathode of V-5 to nearly the grid potential, the thyratron fires, recharging the capacitor. The upper limit of capacitor voltage is determined by the supply voltage, and the voltage drop in the thyratron. Due to the effect of lead inductance, the capacitor is charged to very nearly the full supply potential. By using a supply voltage of 325 volts, and a working range of capacitor voltage of 175 volts, the size of capacitor required is made quite reasonable.

It has been found that this particular type of sweep circuit is quite easy to synchronize. A synchronization terminal has been provided through which the signal being viewed on the oscilloscope may be fed to the grid of V-5. The values of R-10, R-11 and R-12 are reasonably high, so, unless the signal is provided by a high-impedance source, this circuit will have little or no effect on the signal. If difficulties are encountered due to this, it is possible to insert a resistance in series with the synchronization terminal since the synchronizing signal required is quite small.

As has been stated, it is desirable to provide a deflection voltage which is balanced. To accomplish this, the saw-tooth generator is provided with an amplifier which gives a symmetrical output voltage when driven by a signal which is unbalanced with respect to ground. In effect it is a form of self-balancing phase inverter. With the coupling system used, the response of this amplifier is flat down to zero frequency. Leakage currents through the capacitors C-4 and C-5, however, may cause a falling off in response at frequencies of the order of a small fraction of a cycle per second.

The coupling system itself is somewhat unique and is worthy of further consideration. The advantage of employing a large working range of capacitor voltage has been pointed out, as has the necessity for providing a balanced deflection voltage. It is impossible to apply to the grid of the amplifier tube a signal of the order of magnitude of 175 volts peak to peak without resorting to the variety of tube commonly used in transmitters. This is impracticable, and the alternative is to reduce the amplitude of the signal. This is most readily accomplished by means of a voltage divider. A resistive voltage-dividing network connected across the sweep capacitor, however, would greatly increase the leakage current with the resulting non-linearity of sweep voltage. Instead, a capacitor voltage divider consisting of C-4 and C-5, was used. This offers the additional advantage of blocking out the d-c component of the voltage across the sweep capacitor.

When a voltage is applied to two capacitors in series, the voltage division between the two capacitors after a long period of time is determined by the relative values of leakage current through the two capacitors. This means that the average potential at the grid of the amplifier tube, V-7, would vary unless the leakage resistances of C-4 and C-5 were equal, and, in all probability, they are not. To avoid the undesirable effect of the trace slowly drifting to the left or right on the screen of the cathode-ray tube, V-6, a type 6H6 double diode, was connected in the circuit as shown in the schematic diagram. If the minimum voltage across C-4 drops below a certain value determined by the setting of R-15, the right diode conducts, placing a positive charge on C-4. On the other hand, if the maximum voltage across C-4 exceeds a value which is determined by the setting of R-18, the left section of V-6 conducts, applying a negative charge to C-4. Thus, with the proper adjustment of R-15 and R-18, any drift in the average potential on C-4 is compensated for as it occurs.

It will now be shown that, except for the very low frequencies at which leakage currents through C-4 and C-5 may produce distortion, the coupling network is effective for all frequencies. The impedance of a capacitor at any frequency below self-resonance is a pure reactance if the leakage resistance is sufficiently high that it can be neglected. For high-quality mica capacitors, this is certainly true. This reactance may be expressed as:

$$X_c = \frac{1}{2\pi fC} \tag{11}$$

By the potentiometer rule, the voltage appearing across C-4 is:

$$e_4 = \frac{X_{C4}}{X_{C_4} + X_{C_5}} e = \frac{C_5}{C_4 + C_5} e \tag{12}$$

$e_4$ = voltage appearing across C-4.

$e$ = voltage across series combination of C-4 and C-5.

$C_4$ = capacitance of C-4.

$C_5$ = capacitance of C-5.

As is indicated by Equation 12, the voltage appearing across C-4 is independent of frequency, and depends only upon the applied voltage and the capacitances of C-4 and C-5.

The deflection amplifier consists of a type 6SN7 dual triode in a circuit which is frequently used in television circuits to provide a balanced deflection voltage from an unbalanced signal. When a positive signal is applied to the input grid of the amplifier the current through the triode shown on the right in the schematic diagram increases, causing the cathodes of both triodes to become more positive. Since the grid of the left triode is at a fixed potential determined by the setting of R-24, this is effectively a negative signal on the grid of the left triode, causing a decrease in current through the left-hand section of V-7. For perfect balance, the increase in current through the right half of V-7 would have to equal the decrease in current through the left half. This, however, is impossible, since if there is no net change of current through the cathode resistance, R-22, no signal is developed to drive the left section of the amplifier. The ratio of the currents through the two sections of V-7 gives an indication of the unbalance existing in a given circuit. This ratio, as indicated by Equation 8 in the second derivation of Appendix I may be written as:

$$\frac{i_1}{i_2} = 1 + \frac{R_L + R_P}{(\mu+1)\, R_{22}} \tag{13}$$

$i_1$ = alternating component of current through input triode.

$i_2$ = alternating component of current through second triode.

$R_L$ = load resistance of either triode, assuming that the two load resistances are equal.

$R_P$ = dynamic plate resistance of either triode, assuming that both are equal.

$R_{22}$ = cathode resistance.

$\mu$ = amplification factor of each tube, also assumed to be equal.

Equation 13 indicates that the unbalance decreases with increase in the value of cathode resistance, but the extent to which this can be carried is limited by d-c considerations. The value of load resistance is dictated by the required output voltage and the allowable grid swing. This is not serious, however, for considerable unbalance must occur before defocusing of the spot becomes noticeable.

The gain of this type of amplifier may be expressed as:

$$A = \frac{\mu R_L}{R_p + R_L} \tag{14}$$

where symbols are the same as those used in Equation 13. This is seen to be identical to the amplification of a convertional single-stage amplifier, and is in no way affected by the value of the cathode resistance.

In testing the saw-tooth generator, the output from an audio oscillator was applied simultaneously to the vertical-deflection amplifier terminals of two oscilloscopes, one with a medium-persistence and one with a long-persistence screen. The output from the saw-tooth generator was applied directly to the horizontal deflection plates of the cathode-ray tubes in the two oscilloscopes. In this way, it was possible to view the trace on the oscilloscope with the long-persistence screen while photographing the trace on the screen of the other oscilloscope. Synchronization was provided by applying to the synchronization terminal of the saw-tooth generator the output from a pulse generator which was in turn driven by the audio oscillator. This method of synchronization was employed to insure that the trace would be absolutely stable since, in some cases, it was necessary in the photographing process to use exposures up to 10 seconds. The actual apparatus used appears in the List of Apparatus and is shown in Photograph XIII.

The final product of this development, a saw-tooth generator capable of covering the range of frequencies from one cycle per second to thirty cycles per second, is shown in Photograph XIV. An indication of the capabilities of the saw-tooth generator is given by Photographs I through XII. The amount of non-linearity present is most readily seen by reference to Photograph III. It will be noticed that the non-linearity occurs in the early part of the sweep. This is due partly to distortion in the deflection amplifier and partly to the presence of C-1 in the circuit of the saw-tooth generator. The distortion due to the deflection amplifier can be reduced by employing a tube which permits a larger grid swing, and that due to C-1 can be eliminated by removing C-1 and R-13 and placing a large inductance in series with each sweep capacitor. The non-linearity present, however, is not objectionable as is indicated in the photographs of the other traces. Also, the addition of the inductance in series with the sweep capacitor would lengthen the time of retrace which, with the circuit used, is quite short, as can be seen from Photographs X, XI, and XII.

## III. CONCLUSIONS

This investigation shows conclusively that it is possible to build a saw-tooth generator employing a relatively simple circuit for the purpose of providing a time base for a cathode-ray oscilloscope and capable of operating at frequencies well below those at which the sweep in the ordinary oscilloscope becomes useless. The upper frequency limit is of the order of several kilocycles per second, and the range can be extended downward as far as desired by the simple expedient of connecting larger values of sweep capacitance into the circuit.

BIBLIOGRAPHY

Clarke, Arthur C., "Linearity Circuits," Wireless Engineer, No. 249, Vol. XXI, June, 1944, pp. 256-266.

Folkerts, H. F. and P. A. Richards, "Photography of Cathode-Ray Tube Traces," R.C.A. Review, No. 2, Vol. VI, October, 1941, pp. 234-244.

Haller, C. E., "A Linear Timing Axis for Cathode Ray Oscillographs," Review of Scientific Instruments, Vol. IV, July, 1944, pp. 385-386.

Hill, W. R., Jr., "Analysis of Current Stabilizer Circuits," Proceedings of the Institute of Radio Engineers, No. 11, Vol. XXXIII, November, 1945, pp. 785-792.

Neustadt, H., "Thyratron Linear Time Axis for Cathode Ray Oscilloscope," Electronics, June, 1932, pp. 198-199.

Owen, Robert P., "Linear Sweep Circuits," Electronics, December, 1946, pp. 136-138.

Puckle, O. S., Time Bases, John Wiley & Sons, Inc., New York, 1943, 197 pages.

Reich, H. J., Theory and Applications of Electron Tubes, McGraw-Hill Book Company, Inc., New York, 1944, pp. 448-473.

# APPENDIX I

# DERIVATIONS

DERIVATION I

An expression for the change in capacitor discharge current is derived in terms of the change in capacitor voltage, the amplification of the feedback stage, V-1, the amplification of the discharge tube stage, V-2, considered as a cathode follower, the amplification factor of V-2 and the load resistance in the cathode of V-2.

Definitions:

$e$ = change in capacitor voltage.

$i_1$ = change in capacitor discharge current.

$\mu_1$ = amplification factor of the tube V-1.

$\mu_2$ = amplification factor of the tube V-2.

$R_{p1}$ = dynamic plate resistance of V-1.

$R_{p2}$ = dynamic plate resistance of V-2.

$R_{L2} = R_7 + R_8$ = resistance in cathode circuit of the discharge tube V-2.

$A_1$ = amplification of the feedback stage, V-1.

$A_2$ = amplification of the discharge tube stage considered as a conventional cathode follower.

From the equivalent circuit of Diagram I-B, the following equations can be written:

$$i_1R_{L_2} + i_1R_{p2} = e - \mu_2 e_3 \qquad (1)$$

$$e_1 = i_1R_{L2} \qquad (2)$$

$$i_2R_{p1} + i_3R_2 = \mu_1 e_1 \qquad (3)$$

$$i_2R_{p1} + i_4R_9 = \mu_1 e_1 \qquad (4)$$

$$e_2 = i_3R_2 = i_4R_9 \tag{5}$$

$$i_2 = i_3 + i_4 \tag{6}$$

$$e_3 = e_1 + e_2 \tag{7}$$

Since $i_1$ represents the change in discharge current of the capacitor, this is the quantity which is to be determined.

Substituting (7) in (1),

$$i_1R_{L2} + i_1R_{p2} = e - \mu_2e_1 - \mu_2e_2 \tag{8}$$

Substituting (2) and (5) in (8),

$$i_1 \left(R_{L2} + R_{p2} + \mu_2R_{L2}\right) = e - \mu_2R_2i_3 \tag{9}$$

Substituting (6) in (3) and (4) and rearranging,

$$i_3 \left(R_{p1} + R_2\right) + i_4R_{p1} = \mu_1e_1 \tag{10}$$

$$i_3R_{p1} + i_4 \left(R_{p1} + R_9\right) = \mu_1e_1 \tag{11}$$

Solving (10) for $i_4$,

$$i_4 = \frac{\mu_1e_1 - i_3(R_{p1} + R_2)}{R_{p1}} \tag{12}$$

Substituting (12) in (11) and solving for $i_3$,

$$i_3 = \frac{\mu_1e_1R_9}{R_{p1}R_2 + R_{p1}R_9 + R_2R_9} \tag{13}$$

But, from (2) $e_1 = i_1R_{L2}$

Hence:

$$i_3 = \frac{\mu_1 i_1 R_{L2} R_9}{R_{p1}R_2 + R_{p1}R_9 + R_2R_9} \tag{14}$$

Substituting (14) in (9),

$$i_1 (R_{L2} + R_{p2} + \mu_2 R_{L2}) = e - \mu_2 R_{L2} i_1 \frac{\mu_1 R_2 R_9}{R_{p1}R_2 + R_{p1}R_9 + R_2R_9} \tag{15}$$

It can easily be shown that the amplification of the V-1 stage, $A_1$ is:

$$A_1 = \frac{\mu_1 R_2 R_9}{R_{p1}R_2 + R_{p1}R_9 + R_2R_9} \tag{16}$$

Substituting (16) in (15) and solving for $i_1$,

$$i_1 = \frac{e}{\mu_2 R_{L2} A_1 + \left[(\mu_2 + 1) R_{L2} - R_{p2}\right]} \tag{17}$$

Refer to Diagram I-C. The following equations can be written:

$$e_3 = e_i - e_o \tag{18}$$

$$\frac{e_o}{e_i} = A_2 \tag{19}$$

$$i_1 R_{L2} + i_1 R_{p2} = \mu_2 e_3 \tag{20}$$

$$e_o = i_1 R_{L2} \tag{21}$$

From (20)

$$i_1 = \frac{\mu_2 e_3}{R_{L2} + R_{p2}} \tag{22}$$

Substituting (22) in (21):

$$e_o = \frac{\mu_2 e_3 R_{L2}}{R_{L2} + R_{p2}} \tag{23}$$

Substituting (18) in (23) and rearranging:

$$e_o(R_{L2} + R_{p2}) = \mu_2 R_{L2} e_i - \mu_2 R_{L2} e_o \tag{24}$$

$$\frac{e_o}{e_i} = \frac{\mu_2 R_{L2}}{(\mu_2 + 1)R_{L2} + R_{p2}} = A_2 \tag{25}$$

Dividing numerator and denominator of (17) by $\mu_2 R_{L2}$,

$$i_1 = \frac{\dfrac{e}{\mu_2 R_{L2}}}{A_1 + \dfrac{(\mu_2 + 1)R_{L2} + R_{p2}}{\mu_2 R_{L2}}} \tag{26}$$

Substituting (25) in (26) and rearranging:

$$i_1 = \frac{eA_2}{\mu_2 R_{L2}(A_1 A_2 + 1)} \tag{27}$$

# DERIVATION II

Two expressions are derived, one of which gives the amplification of the single-ended input, push-pull output amplifier in terms of the amplification factor and plate resistance of the tube, and the load resistance, the other giving the ratio of the two alternating components of plate current, an indication of the degree of unbalance of the stage.

Refer to Diagram II-B. Since the push-pull deflection amplifier consists of the two units of a 6SN7 dual triode, the following assumption can be made without introducing appreciable error:

$R_{p1} = R_{p2} = R_p$ = dynamic plate resistance of each section tube.

$\mu_1 = \mu_2 = \mu$ = amplification factor of each tube section.

Furthermore, since in the usual case, the two load resistors are equal, let:

$R_{20} = R_{21} = R_L$

With the above simplifications, the following equations can be written from the equivalent circuit:

$$e = e_1 + e_2 \tag{1}$$

$$e_2 = (i_1 - i_2)\, R_{22} \tag{2}$$

$$e_o = (i_1 + i_2)\, R_L \tag{3}$$

$$i_1\,(R_L + R_p + R_{22}) - i_2\, R_{22} = \mu\, e_1 \tag{4}$$

$$-i_1\, R_{22} + i_2\,(R_L + R_p + R_{22}) = \mu\, e_2 \tag{5}$$

Substituting (2) in (5) and rearranging,

$$i_1\ (\mu + 1)\ R_{22} - i_2 \left[(\mu + 1)\ R_{22} + R_L + R_p\right] = 0 \tag{6}$$

$$i_1 = i_2 \quad \frac{(\mu + 1)\ R_{22} + R_L + R_p}{(\mu + 1)\ R_{22}} \tag{7}$$

$$\frac{i_1}{i_2} = 1 + \frac{R_L + R_p}{(\mu + 1)\ R_{22}} \tag{8}$$

Substituting (1) in (4)

$$i_1\ (R_L + R_p + R_{22}) - i_2 R_{22} = \mu e - \mu e_2 \tag{9}$$

Substituting (2) in (9) and rearranging,

$$i_1 = \frac{\mu e + i_2(\mu + 1)\ R_{22}}{(\mu + 1)\ R_{22} + R_L + R_p} \tag{10}$$

Equating (10) and (7),

$$\frac{i_2 \left[(\mu + 1)\ R_{22} + R_L + R_p\right]}{(\mu + 1)\ R_{22}} = \frac{\mu e + i_2\ (\mu + 1)\ R_{22}}{(\mu + 1)\ R_{22} + R_L + R_p} \tag{11}$$

Cross multiplying and solving for $i_2$,

$$i_2 = \frac{\mu e\ (\mu + 1)\ R_{22}}{(R_L + R_p)^2 + 2\ (R_L + R_p)\ (\mu + 1)\ R_{22}} \tag{12}$$

$$i_1 = \frac{\mu e \left[\ (\mu + 1)\ R_{22} + R_L + R_p\right]}{(R_L + R_p)^2 + 2\ (R_L + R_p)\ (\mu + 1)\ R_{22}} \tag{13}$$

From (3),

$$e_o = (i_1 + i_2)\ R_L$$

Substituting (12) and (13) in this expression,

$$\frac{e_o}{e} = \frac{\mu \left[ R_L + R_p + 2\,(\mu + 1)\, R_{22} \right] R_L}{(R_1 + R_p)\,\left[ R_L + R_p + 2\,(\mu + 1)\, R_{22} \right]} \qquad (14)$$

Since the amplification, A, of the stage is

$$A = \frac{e_o}{e},$$

From (14),

$$A = \frac{\mu R_L}{R_p + R_L} \qquad (15)$$

# APPENDIX II

## TABLES

TABLE I

Capacitor Discharge Current, $I_c$, in milliamperes corresponding to values of Capacitor Voltage, $V_c$, in volts.

The discharge circuit is the same as the circuit shown on the schematic diagram except:

Screen of V-2 returned to B+ through 68000 ohms.
Cathode of V-4 returned to -250 volts through 45000 ohms.
Cathode of V-1 tied to ground through 2200 ohms.

Values are plotted as Curve I-A.

| $V_c$ | $I_c$ | $V_c$ | $I_c$ |
|---|---|---|---|
| 0 | 0.00 | 180 | 4.16 |
| 10 | 0.00 | 190 | 4.17 |
| 20 | 0.00 | 200 | 4.17 |
| 30 | 0.26 | 210 | 4.18 |
| 40 | 1.35 | 220 | 4.19 |
| 50 | 3.37 | 230 | 4.19 |
| 60 | 3.71 | 240 | 4.20 |
| 70 | 3.87 | 250 | 4.20 |
| 80 | 3.96 | 260 | 4.20 |
| 90 | 4.01 | 270 | 4.20 |
| 100 | 4.03 | 280 | 4.20 |
| 110 | 4.05 | 290 | 4.21 |
| 120 | 4.07 | 300 | 4.21 |
| 130 | 4.09 | 320 | 4.21 |
| 140 | 4.11 | 340 | 4.22 |
| 150 | 4.12 | 360 | 4.23 |
| 160 | 4.14 | 380 | 4.23 |
| 170 | 4.15 | 400 | 4.23 |

TABLE II

Capacitor Discharge Current, $I_c$, in milliamperes, corresponding to values of Capacitor Voltage, $V_c$, in volts. The discharge circuit is the same as for Table I, except the resistor in the cathode of V-2 was changed to 8000 ohms. Values are plotted as Curve I-B.

| $V_c$ | $I_c$ | $V_c$ | $I_c$ |
|---|---|---|---|
| 0 | 0.00 | 180 | 2.70 |
| 10 | 0.00 | 190 | 2.70 |
| 20 | 0.00 | 200 | 2.70 |
| 30 | 0.00 | 210 | 2.71 |
| 40 | 1.50 | 220 | 2.71 |
| 50 | 2.10 | 230 | 2.71 |
| 60 | 2.36 | 240 | 2.72 |
| 70 | 2.49 | 250 | 2.72 |
| 80 | 2.55 | 260 | 2.72 |
| 90 | 2.60 | 270 | 2.73 |
| 100 | 2.61 | 280 | 2.73 |
| 110 | 2.63 | 290 | 2.73 |
| 120 | 2.64 | 300 | 2.73 |
| 130 | 2.66 | 320 | 2.74 |
| 140 | 2.67 | 340 | 2.74 |
| 150 | 2.68 | 360 | 2.75 |
| 160 | 2.69 | 380 | 2.75 |
| 170 | 2.69 | 400 | 2.76 |

TABLE III

Capacitor Discharge Current, $I_c$, in milliamperes, corresponding to values of Capacitor Voltage, $V_c$, in volts. Discharge circuit same as circuit shown on Schematic Diagram, except:

Cathode of V-1 returned to ground through 2200 ohms.
Screen of V-2 returned to B+ through 68000 ohms.

Values are plotted as Curve I-C.

| $V_c$ | $I_c$ | $V_c$ | $I_c$ |
|---|---|---|---|
| 0 | 0.00 | 180 | 5.29 |
| 10 | 0.00 | 190 | 5.30 |
| 20 | 0.00 | 200 | 5.31 |
| 30 | 0.20 | 210 | 5.32 |
| 40 | 1.00 | 220 | 5.33 |
| 50 | 2.10 | 230 | 5.33 |
| 60 | 3.80 | 240 | 5.34 |
| 70 | 4.95 | 250 | 5.34 |
| 80 | 5.00 | 260 | 5.35 |
| 90 | 5.07 | 270 | 5.35 |
| 100 | 5.13 | 280 | 5.35 |
| 110 | 5.16 | 290 | 5.36 |
| 120 | 5.20 | 300 | 5.37 |
| 130 | 5.22 | 320 | 5.38 |
| 140 | 5.22 | 340 | 5.38 |
| 150 | 5.24 | 360 | 5.39 |
| 160 | 5.27 | 380 | 5.39 |
| 170 | 5.28 | 400 | 5.40 |

TABLE IV

Capacitor Discharge Current, $I_c$, in milliamperes, corresponding to Capacitor Voltages, $V_c$, from 0 to 450 volts, for values of cathode resistance, R-7 + R-8, in the circuit of the discharge tube, V-2. Values are plotted in Curves II and III. The circuit is that shown on the Schematic Diagram.

| | R-7 + R-8 Ohms x $10^4$ | | | | |
|---|---|---|---|---|---|
| | 1.5 | 1.83 | 2.19 | 3.63 | 7.27 |
| $V_c$ | $I_c$ | $I_c$ | $I_c$ | $I_c$ | $I_c$ |
| 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.000 |
| 10 | 0.11 | 0.02 | 0.00 | 0.00 | 0.000 |
| 20 | 0.64 | 0.42 | 0.24 | 0.04 | 0.000 |
| 30 | 1.10 | 0.84 | 0.59 | 0.29 | 0.000 |
| 40 | 1.50 | 1.18 | 0.93 | 0.51 | 0.090 |
| 50 | 1.80 | 1.53 | 1.20 | 0.74 | 0.235 |
| 60 | 2.37 | 1.91 | 1.50 | 1.00 | 0.370 |
| 70 | 2.83 | 2.35 | 1.83 | 1.15 | 0.520 |
| 80 | 3.35 | 2.75 | 2.15 | 1.38 | 0.660 |
| 90 | 3.86 | 3.40 | 2.48 | 1.60 | 0.942 |
| 100 | 4.40 | 3.66 | 2.88 | 1.82 | 0.964 |
| 110 | 4.70 | 3.85 | 2.94 | 1.84 | 0.966 |
| 120 | 4.80 | 3.92 | 2.98 | 1.85 | 0.967 |
| 130 | 4.84 | 3.94 | 2.98 | 1.85 | 0.969 |
| 140 | 4.84 | 3.94 | 2.98 | 1.86 | 0.970 |
| 150 | 4.84 | 3.94 | 2.98 | 1.86 | 0.971 |
| 200 | 4.85 | 3.96 | 2.99 | 1.87 | 0.978 |
| 250 | 4.86 | 3.97 | 2.99 | 1.88 | 0.980 |
| 300 | 4.87 | 3.98 | 3.00 | 1.88 | 0.987 |
| 350 | 4.88 | 3.99 | 3.00 | 1.89 | 0.992 |
| 400 | 4.89 | 3.99 | 3.01 | 1.89 | 1.000 |
| 450 | 4.90 | 4.00 | 3.01 | 1.89 | 1.000 |

TABLE V

Values and ratings of component parts shown on Schematic Diagram.

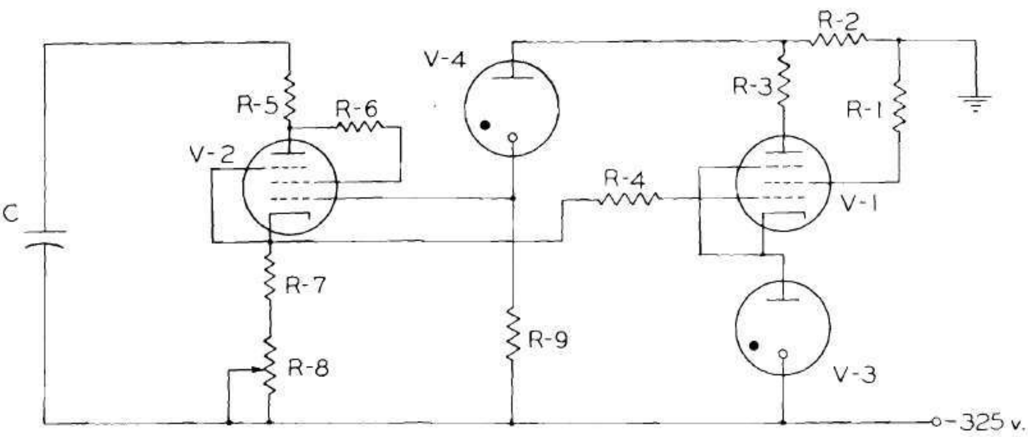
| | | | |
|---|---|---|---|
| C-1 | 0.1 mfd., 450 volts | R-14 | 100000 ohms, 1/2 watt |
| C-2 | 1 mfd., 450 volts | R-15 | 20000 ohms, pot. |
| C-3 | 6 mfd., 450 volts | R-16 | 10000 ohms, 1/2 watt |
| C-4 | 0.01 mfd., 450 volts | R-17 | 100000 ohms, 1/2 watt |
| C-5 | 0.001 mfd., 450 volts | R-18 | 20000 ohms, pot. |
| C-6 | 0.01 mfd., 450 volts | R-19 | 25000 ohms, 1/2 watt |
| R-1 | 22000 ohms, 1 watt | R-20 | 8200 ohms, 1 watt |
| R-2 | 8000 ohms, 5 watt | R-21 | 8200 ohms, 1 watt |
| R-3 | 100 ohms, 1/2 watt | R-22 | 5000 ohms, 1 watt |
| R-4 | 220000 ohms, 1/2 watt | R-23 | 220000 ohms, 1 watt |
| R-5 | 100 ohms, 1/2 watt | R-24 | 20000 ohms, pot. |
| R-6 | 3300 ohms, 1/2 watt | R-25 | 3300 ohms, 1/2 watt |
| R-7 | 12000 ohms, 1 watt | V-1 | 6AC7/1852 |
| R-8 | 70000 ohms, pot. | V-2 | 6AC7/1852 |
| R-9 | 25000 ohms, 1 watt | V-3 | OA3/VR-75 |
| R-10 | 100000 ohms, 1/2 watt | V-4 | OC3/VR-105 |
| R-11 | 20000 ohms, pot. | V-5 | 2051 |
| R-12 | 75000 ohms, 1/2 watt | V-6 | 6H6 |
| R-13 | 300 ohms, 1/2 watt | V-7 | 6SN7 |

# APPENDIX III

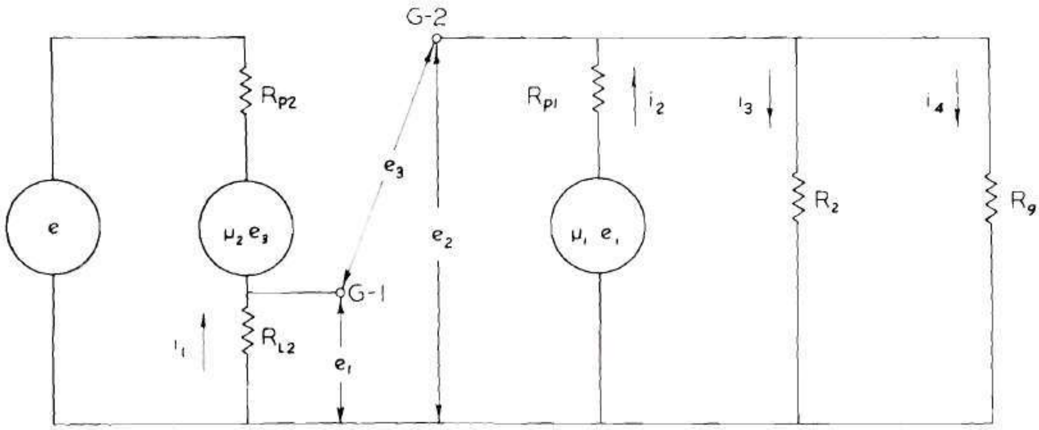
## DIAGRAMS

DIAGRAM I CAPACITOR DISCHARGE CIRCUIT

A. ACTUAL CIRCUIT




B. EQUIVALENT CIRCUIT




C. EQUIVALENT CIRCUIT - CATHODE FOLLOWER

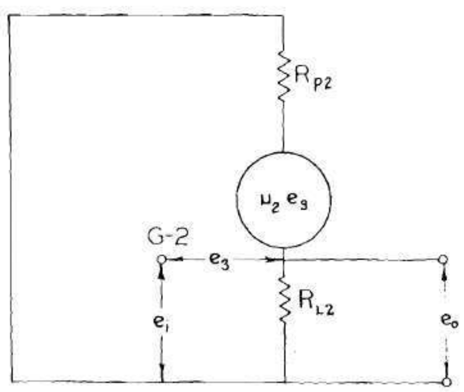
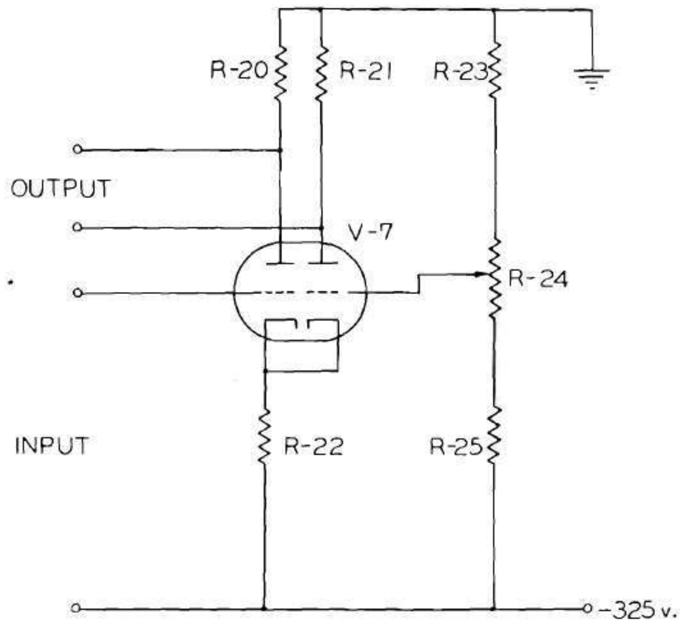
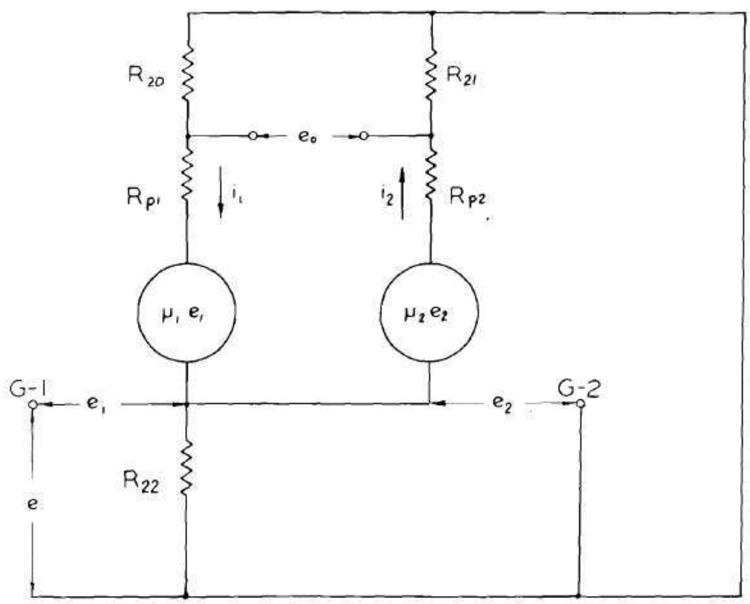


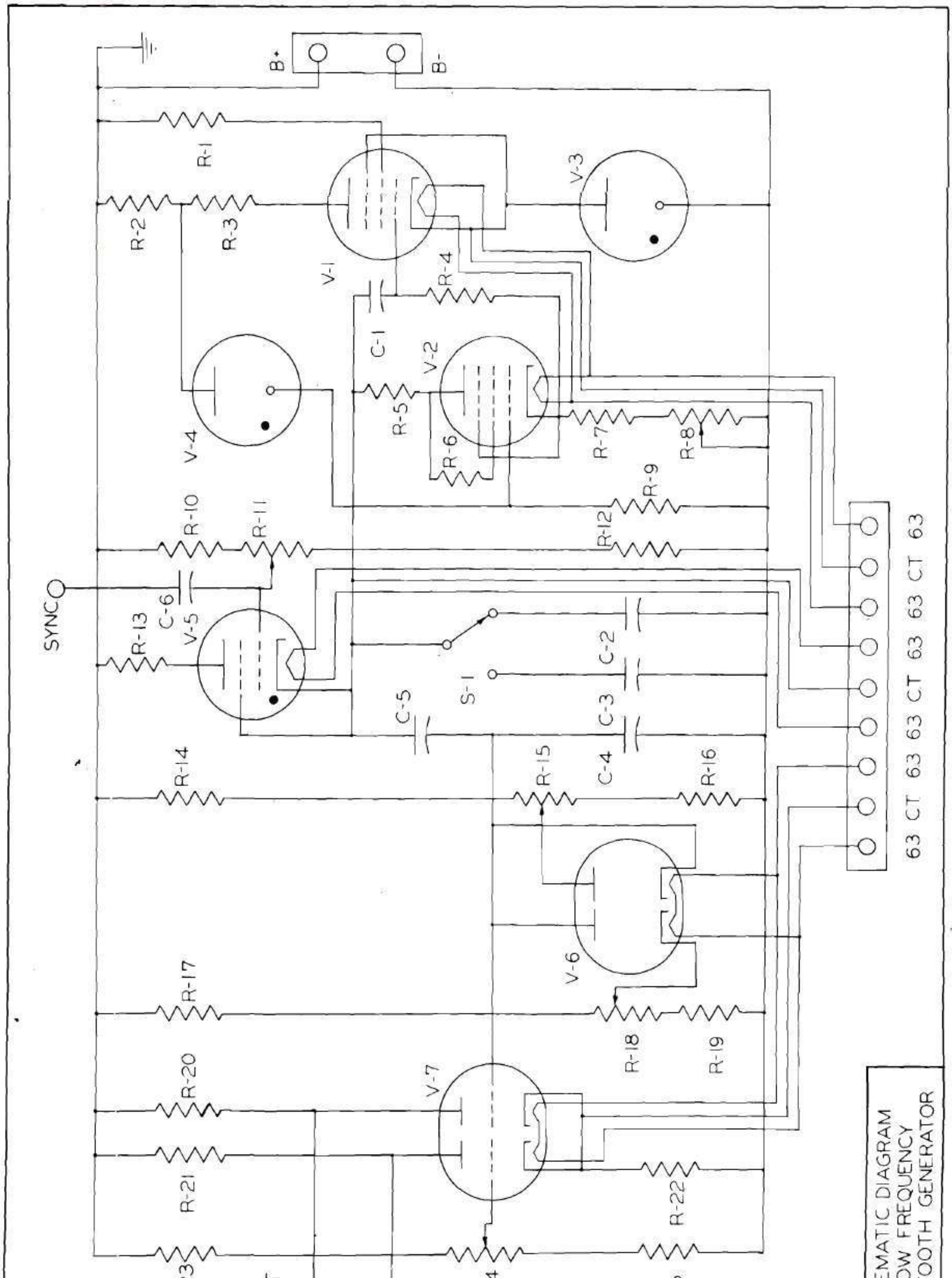
DIAGRAM II PUSH-PULL DEFLECTION AMPLIFIER

A. ACTUAL CIRCUIT




B. EQUIVALENT CIRCUIT

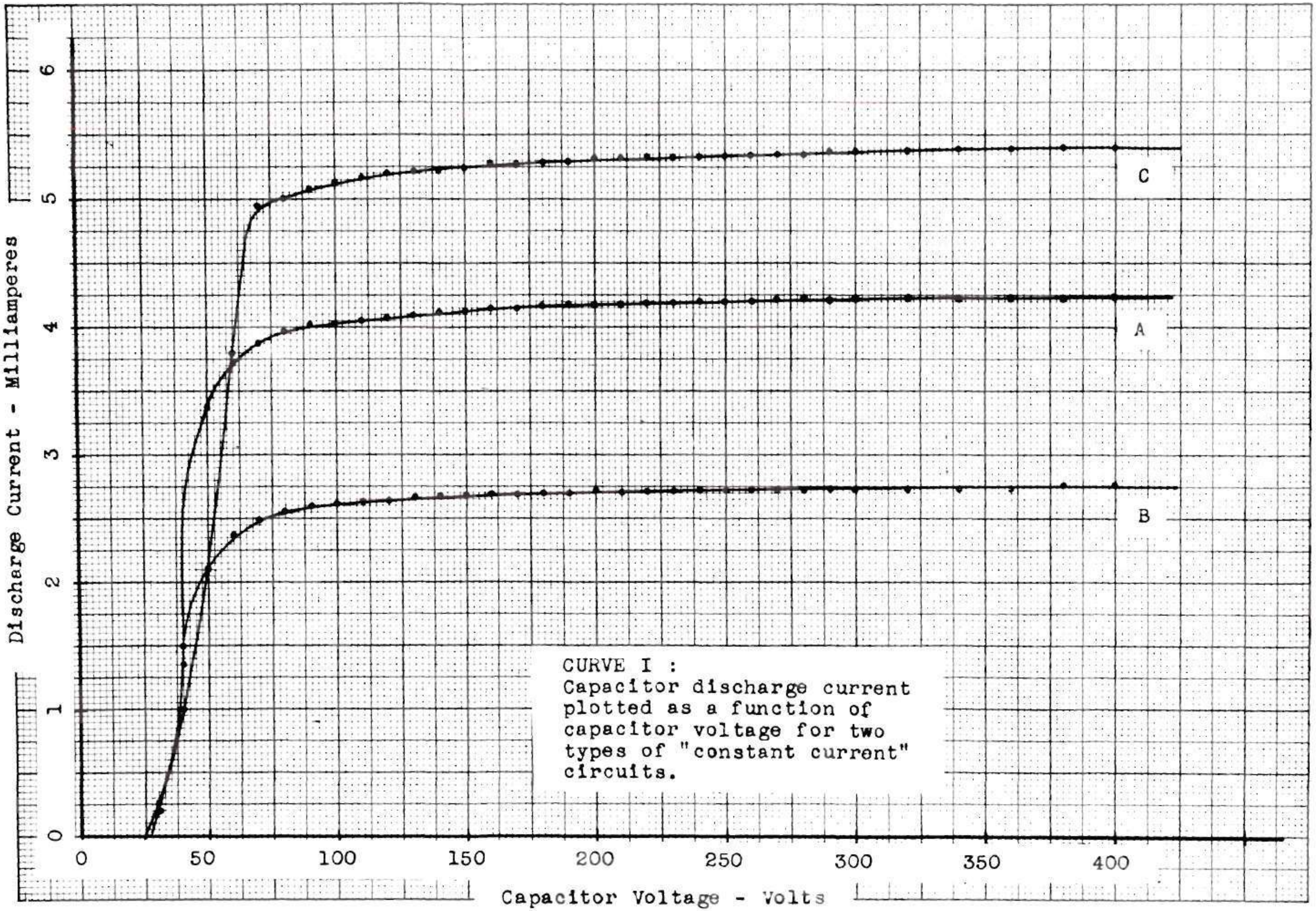




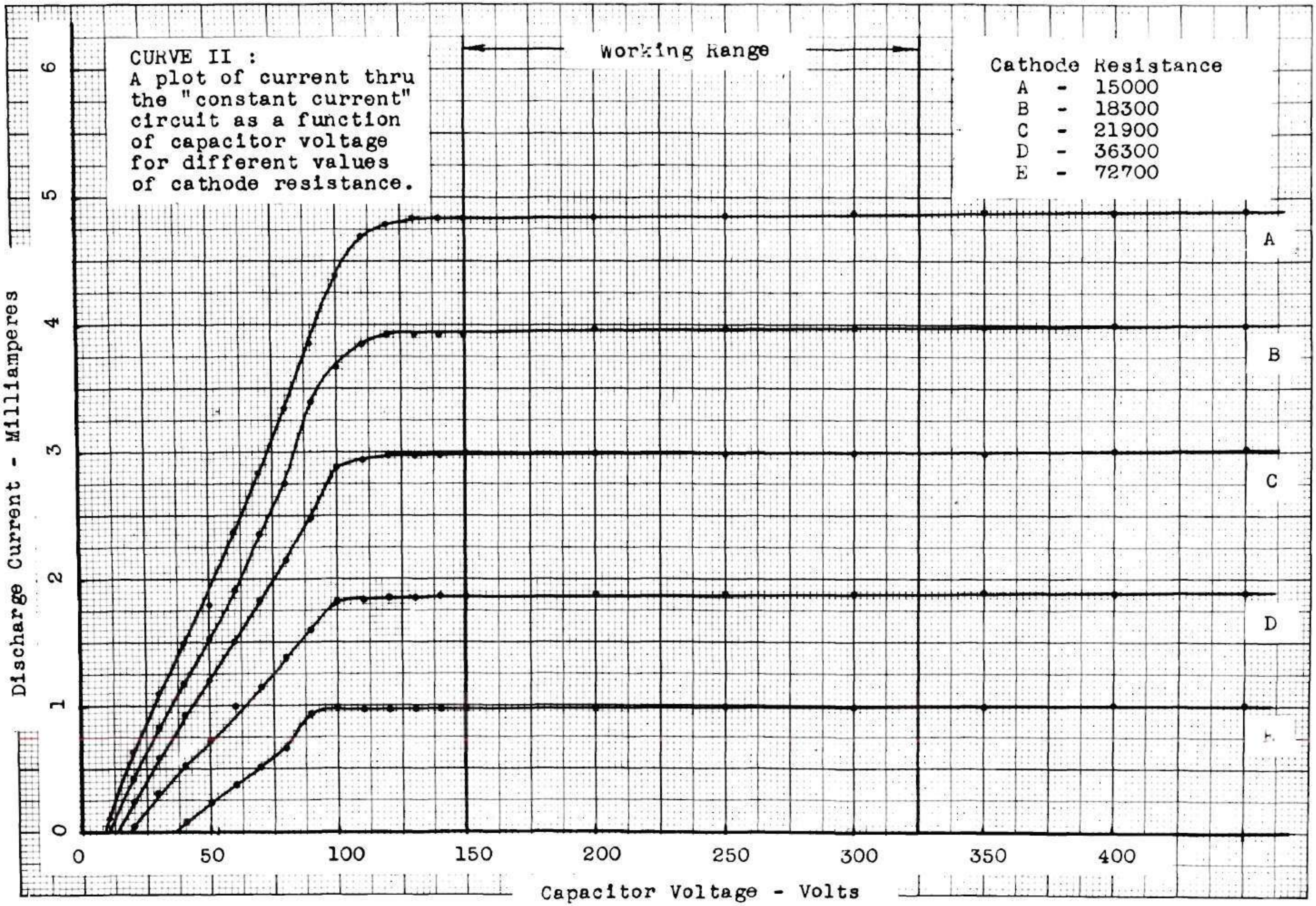

SCHEMATIC DIAGRAM
LOW FREQUENCY
SAWTOOTH GENERATOR

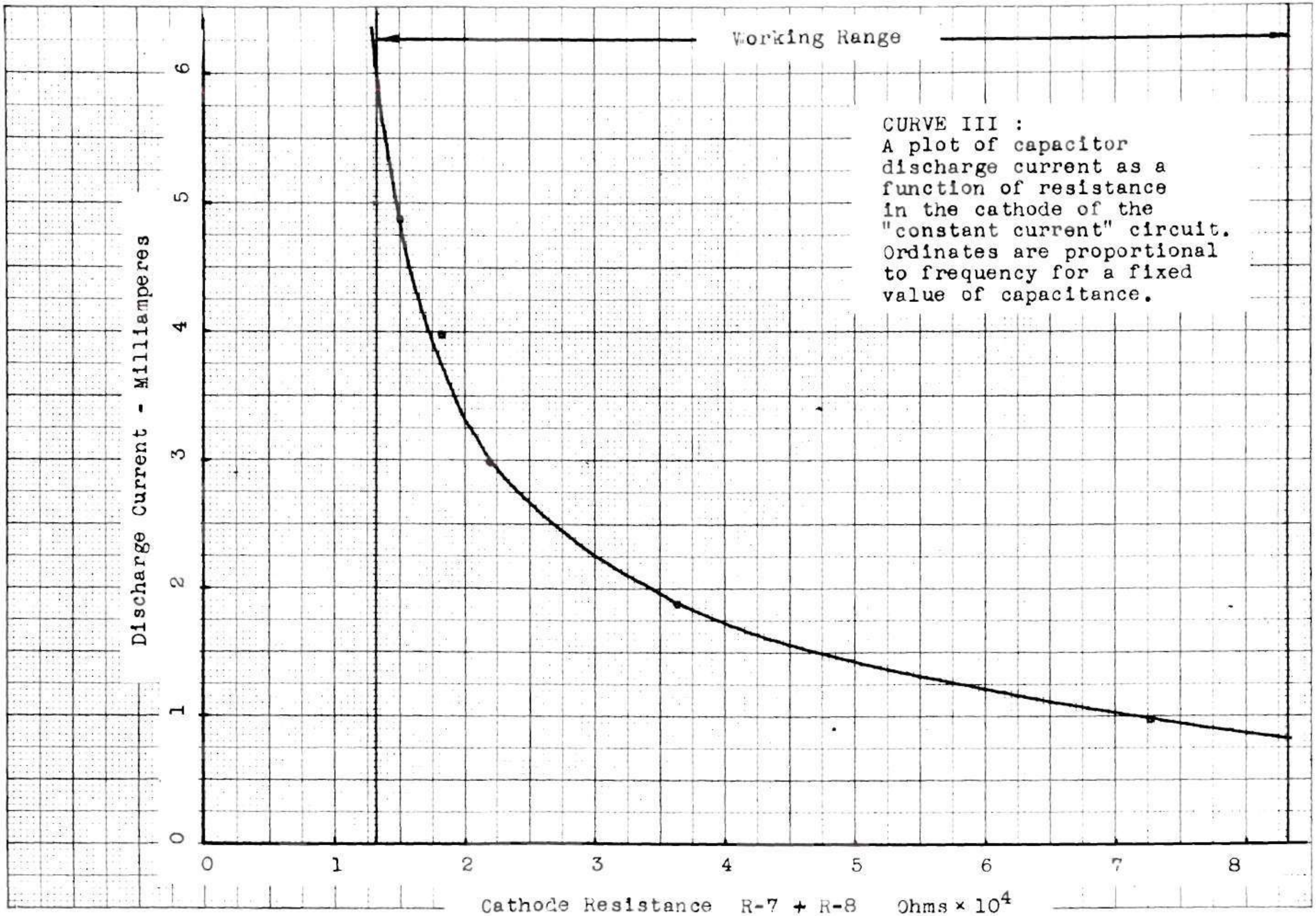
# APPENDIX IV

## CURVES



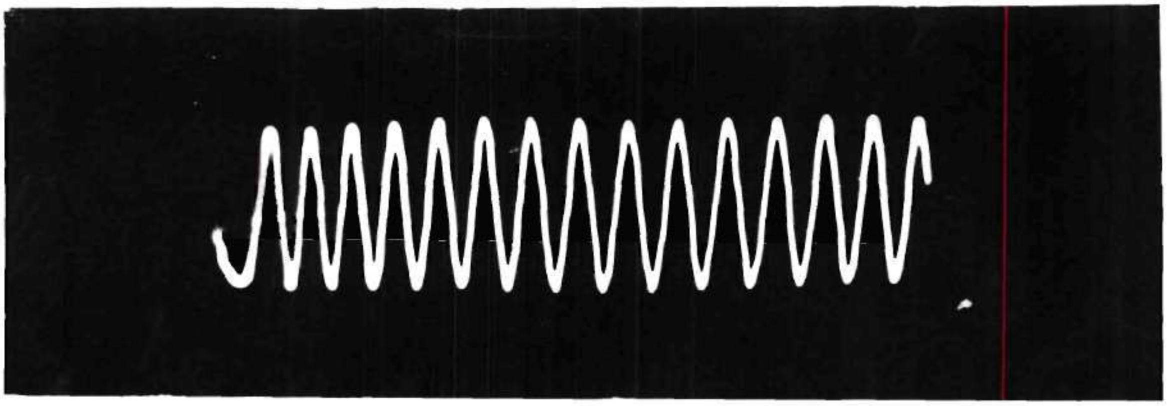
CURVE I :
Capacitor discharge current plotted as a function of capacitor voltage for two types of "constant current" circuits.


CURVE II :
A plot of current thru the "constant current" circuit as a function of capacitor voltage for different values of cathode resistance.
Working Range
Cathode Resistance
A - 15000
B - 18300
C - 21900
D - 36300
E - 72700
Discharge Current - Milliamperes
0
1
2
3
4
5
6
0
50
100
150
200
250
300
350
400
Capacitor Voltage - Volts
A
B
C
D


Working Range
CURVE III :
A plot of capacitor
discharge current as a
function of resistance
in the cathode of the
"constant current" circuit.
Ordinates are proportional
to frequency for a fixed
value of capacitance.
Discharge Current - Milliamperes
0
1
2
3
4
5
6
0
1
2
3
4
5
6
7
8
Cathode Resistance R-7 + R-8 Ohms × $10^4$

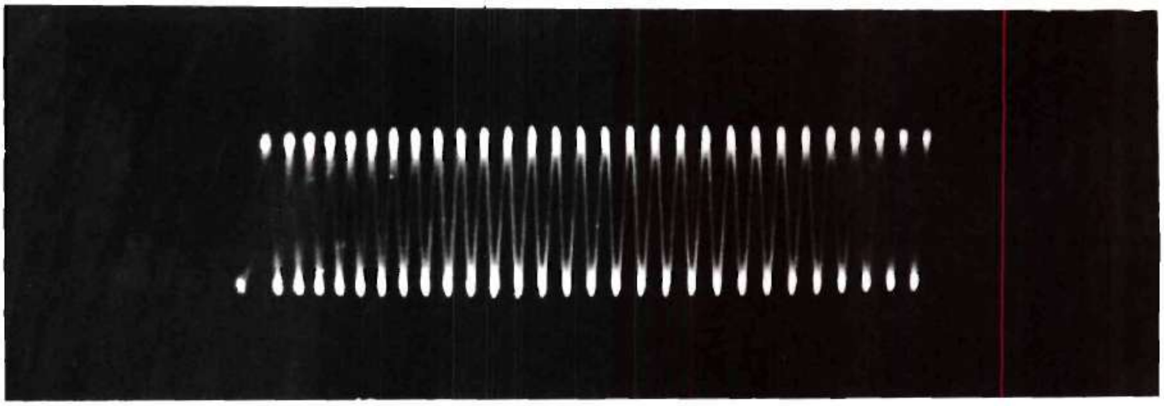
APPENDIX V

PHOTOGRAPHS



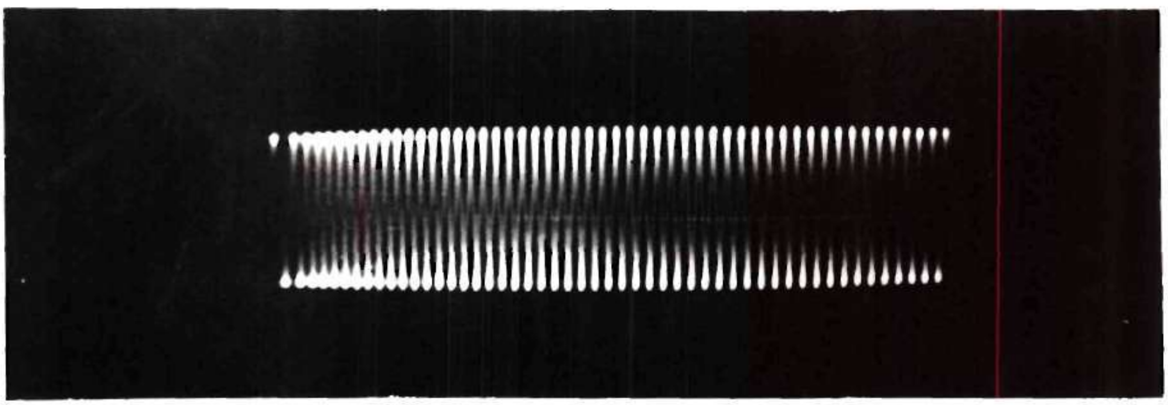
PHOTOGRAPH I : Sweep: 1 c.p.s. - Low Range

Signal: 15 c.p.s.



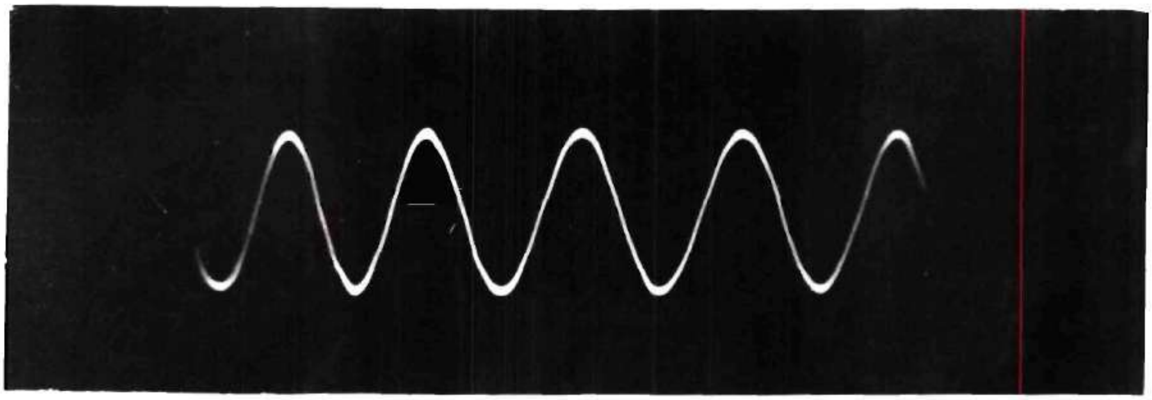
PHOTOGRAPH II : Sweep: 1 c.p.s. - Low Range

Signal: 30 c.p.s.



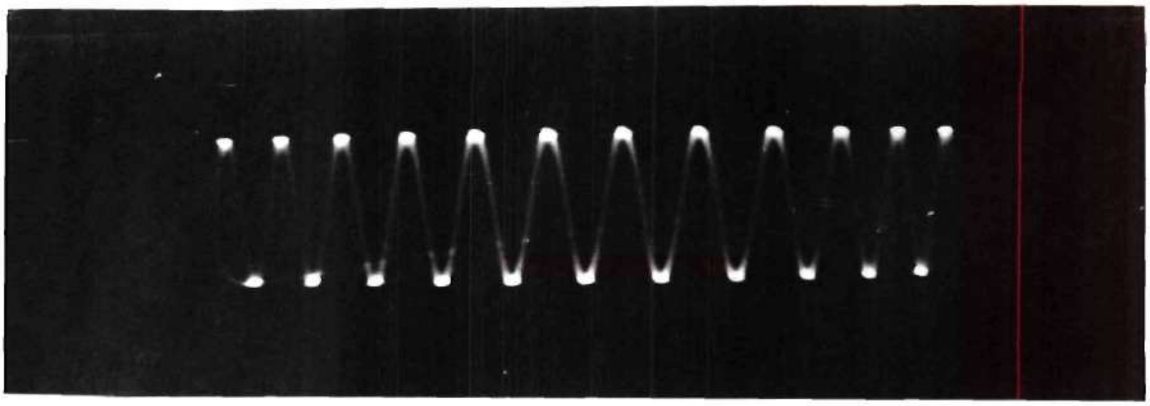
PHOTOGRAPH III : Sweep: 1.2 c.p.s. - Low Range

Signal: 60 c.p.s.



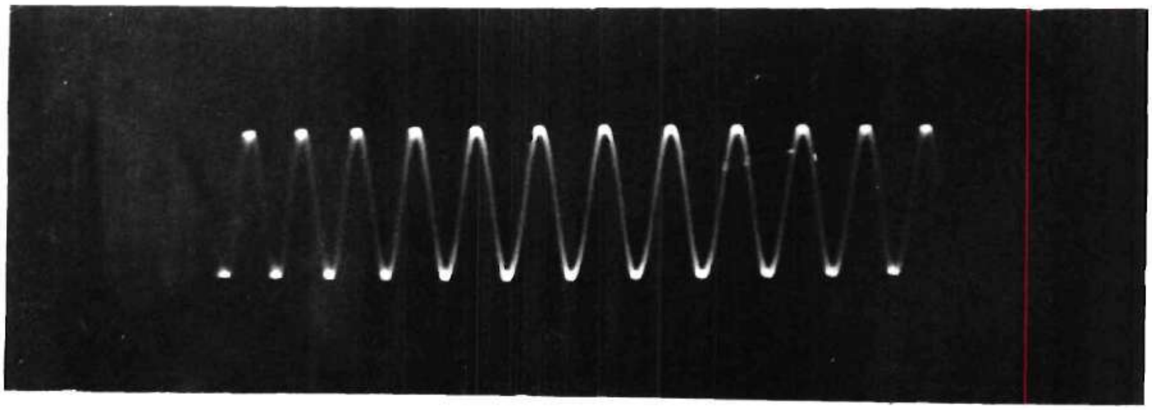
PHOTOGRAPH IV : Sweep: 3 c.p.s. - Low Range

Signal: 15 c.p.s.



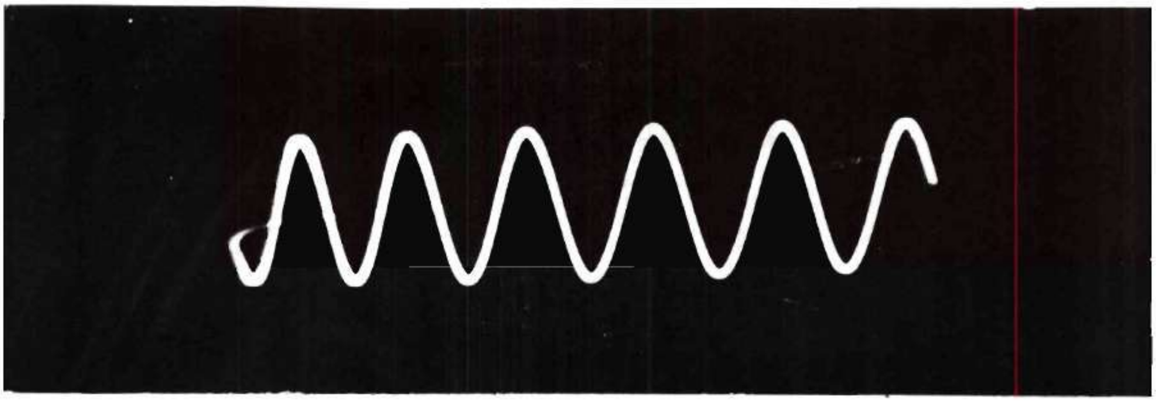
PHOTOGRAPH V : Sweep: 5 c.p.s. - Low Range

Signal: 60 c.p.s.



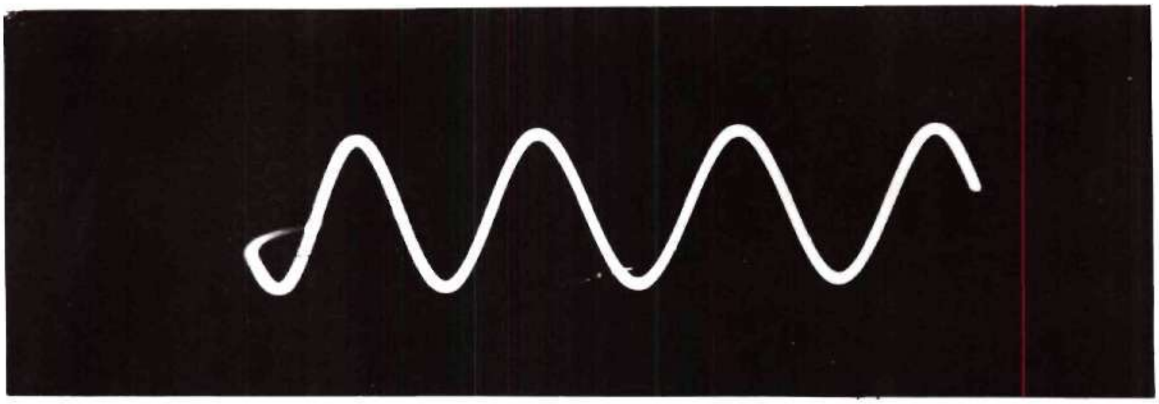
PHOTOGRAPH VI : Sweep: 5 c.p.s. - High Range

Signal: 60 c.p.s.



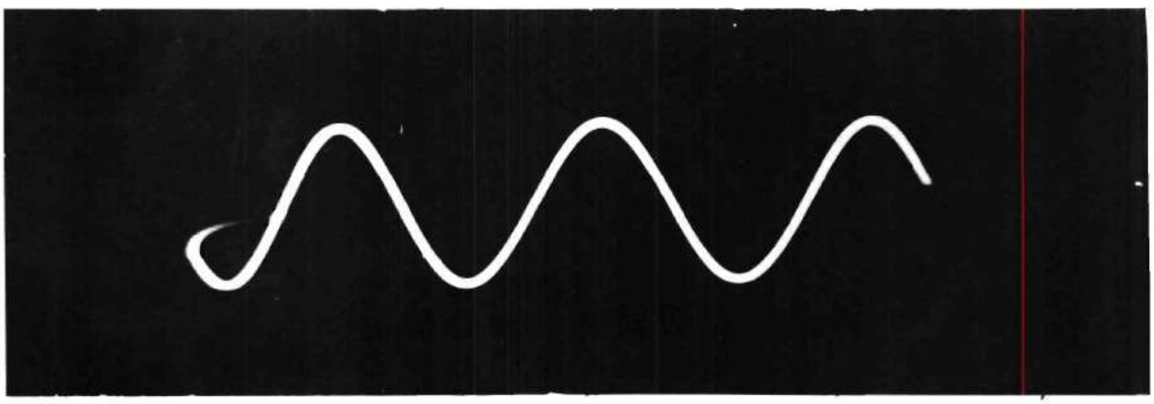
PHOTOGRAPH VII : Sweep: 10 c.p.s. - High Range

Signal: 60 c.p.s.



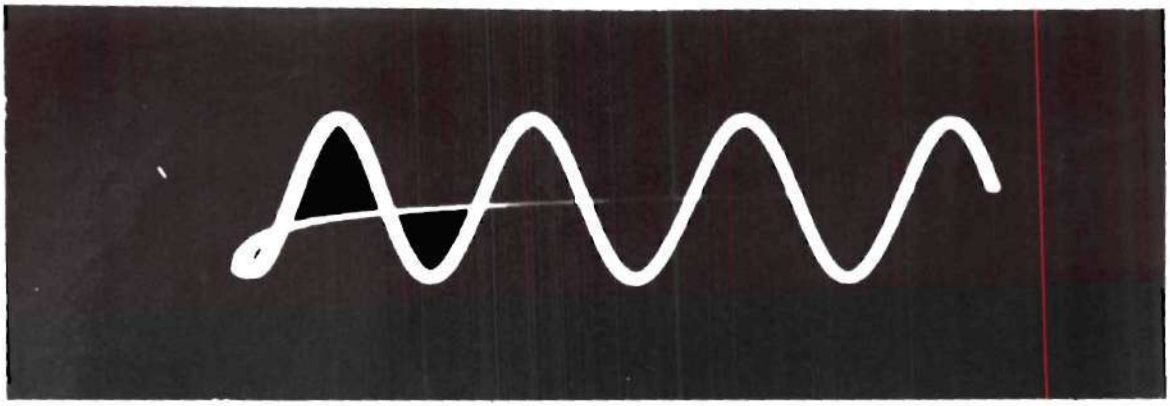
PHOTOGRAPH VIII : Sweep: 15 c.p.s. - High Range

Signal: 60 c.p.s.



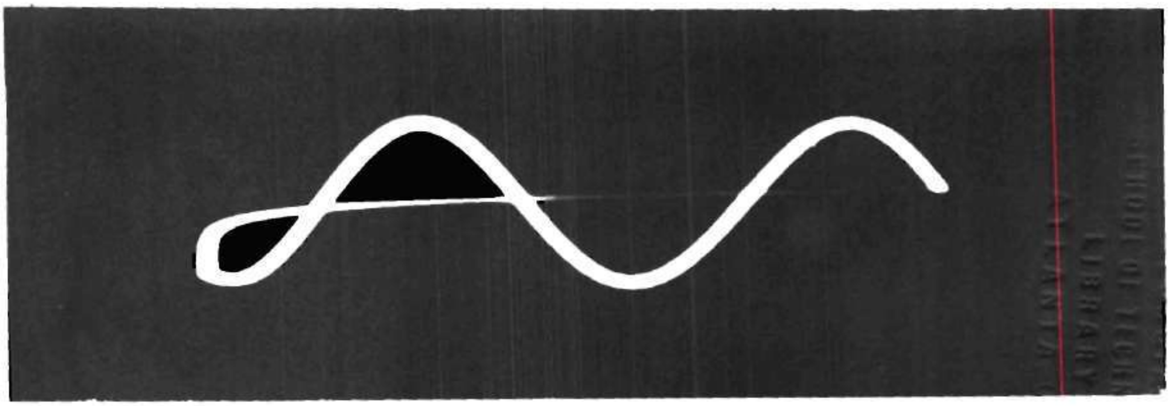
PHOTOGRAPH IX : Sweep: 20 c.p.s. - High Range

Signal: 60 c.p.s.



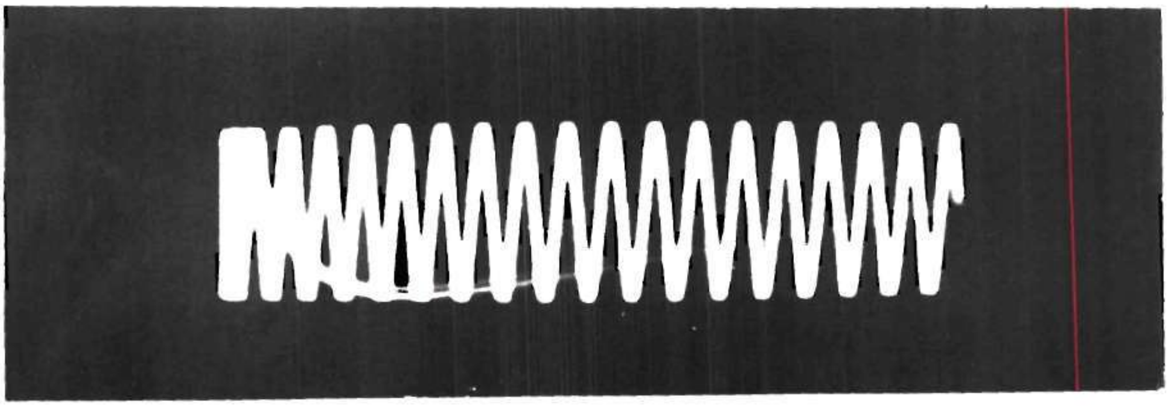
PHOTOGRAPH X : Sweep: 25 c.p.s. - High Range

Signal: 100 c.p.s.



PHOTOGRAPH XI : Sweep: 30 c.p.s. - High Range

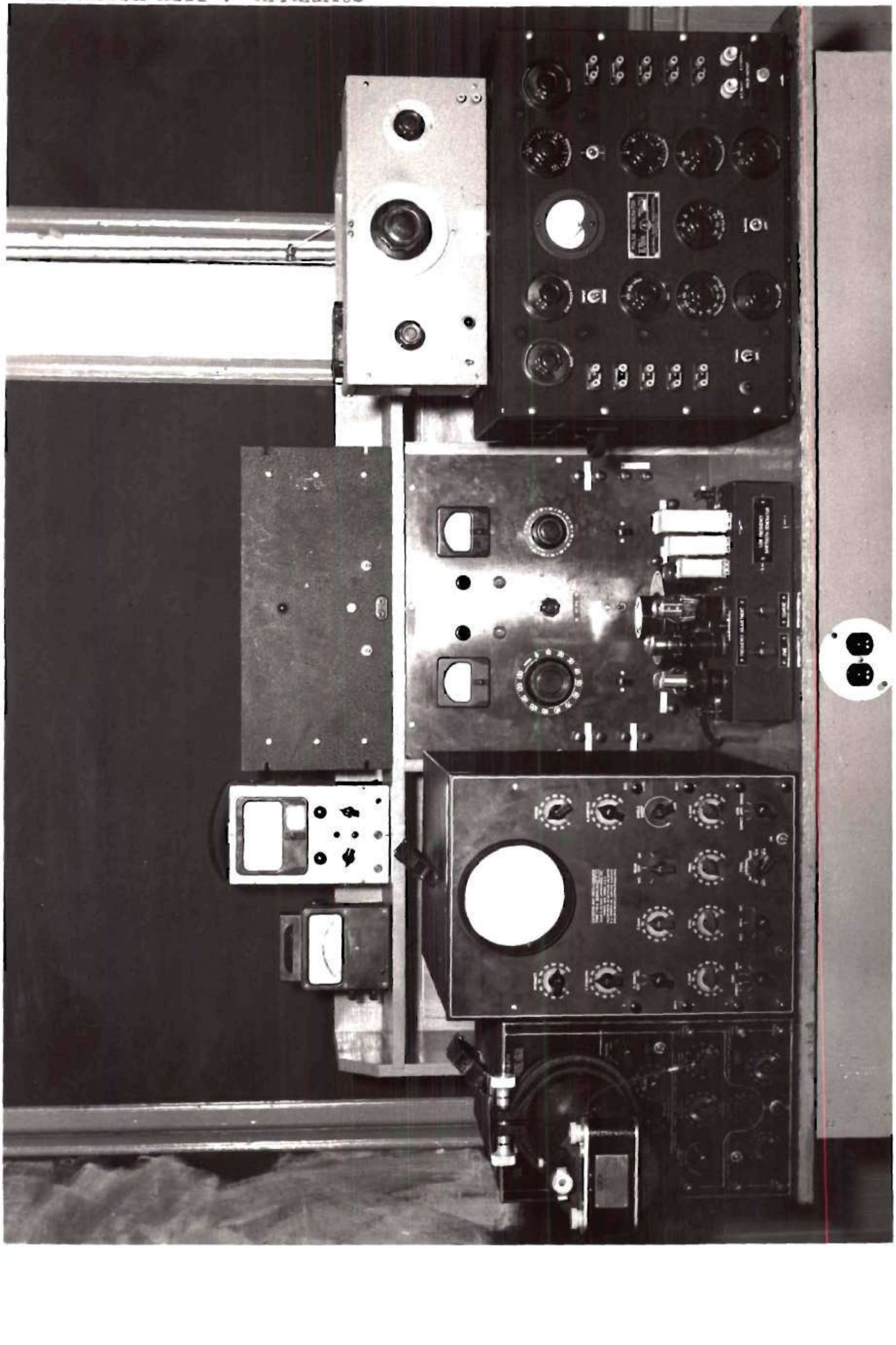
Signal: 60 c.p.s.



PHOTOGRAPH XII : Sweep: 30 c.p.s. - High Range

Signal: 570 c.p.s.

PHOTOGRAPH XIII : APPARATUS



PHOTOGRAPH XIV : LOW FREQUENCY SAW-TOOTH GENERATOR